

Can Galileons support Lorentzian wormholes?

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Abstract

We discuss the possibility of constructing stable, static, spherically symmetric, asymptotically flat Lorentzian wormhole solutions in General Relativity coupled to a generalized Galileon field π . Assuming that Minkowski space-time is obtained at $\partial\pi = 0$, we find that there is tension between the properties of the energy-momentum tensor required to support a wormhole (violation of average null energy conditions) and stability of the Galileon perturbations about the putative solution (absence of ghosts and gradient instabilities). In 3-dimensional space-time, this tension is strong enough to rule out wormholes with above properties. In higher dimensions, including the most physically interesting case of 4-dimensional space-time, wormholes, if any, must have fairly contrived shapes.

1 Introduction and summary

Lorentzian wormholes [1], if existed, would be fascinating objects [2, 3, 4, 5]. In classical General Relativity, however, asymptotically flat Lorentzian wormholes can be supported only by matter that violates the null energy condition, NEC [3, 6, 7], while known forms of matter do not have this property (an interesting example of a Lorentzian wormhole which is not asymptotically flat is given in Ref. [8]). Furthermore, in most of classical field theory models, NEC violation, if any, is plagued by ghosts and/or gradient instabilities. In particular, scalar field theories with the Lagrangians involving at most first space-time derivatives may admit NEC-violating solutions, including Lorentzian wormholes (see, e.g., Ref. [9] and

references therein), but perturbations about these backgrounds have gradient instabilities and/or ghosts [10].

It is known, however, that there exist scalar field theories whose Lagrangians contain second derivative terms, and yet whose field equations are second order [11, 12, 13, 14, 15, 16]. These theories, dubbed generalized Galileon models, admit classical NEC-violating solutions of cosmological type, which do not have obvious pathologies [17, 18, 19, 20] (for a review see, e.g., Ref. [21]), modulo a superluminality issue [22, 23].

It is therefore natural to ask whether generalized Galileon theories admit stable Lorentzian wormholes within General Relativity. It is this question that we address in this paper, albeit not in full generality. The qualifications are as follows. First, we specify to a subclass of generalized Galileons, in which the Lagrangians have most commonly studied form [18, 19]

$$L = F(\pi, X) + K(\pi, X)\Box\pi, \quad (1)$$

where π is the Galileon field, F and K are arbitrary Lagrangian functions, and

$$X = \nabla_\mu \pi \nabla^\mu \pi, \quad \Box\pi = \nabla_\mu \nabla^\mu \pi.$$

We further assume that there is Minkowski limit, which occurs at

$$\partial_\mu \pi = 0.$$

Note that assuming that K is regular at $X = 0$, one can set

$$K(\pi, X = 0) = 0, \quad (2)$$

since the term $K(\pi, 0)\Box\pi$ can be absorbed into F upon integration by parts. Note also that the scalar potential, if any, is contained in F , namely, $V(\pi) = -F(\pi, 0)$.

Second, we consider static and spherically symmetric wormholes in $(d + 2)$ -dimensional space-time. The metric is (signature $(+, -, \dots, -)$)

$$ds^2 = a^2(r)dt^2 - b^2(r)dr^2 - c^2(r)\gamma_{\alpha\beta}dx^\alpha dx^\beta,$$

where x^α and $\gamma_{\alpha\beta}$ are coordinates and metric on unit d -dimensional sphere. The coordinate r runs from $-\infty$ to $+\infty$, and the wormhole geometry is assumed to be asymptotically flat. For $d \geq 2$ (four or more space-time dimensions) this implies the asymptotic behavior

$$d \geq 2 : \quad a \rightarrow a_\pm, \quad b \rightarrow 1, \quad c(r) \rightarrow \pm r, \quad \text{as } r \rightarrow \pm\infty, \quad (3)$$

see Fig. 1, where a_\pm are positive constants.

In 3-dimensional space-time ($d = 1$) the asymptotics of $c(r)$ is less restricted,

$$d = 1 : \quad a \rightarrow a_\pm, \quad b \rightarrow 1, \quad c(r) \rightarrow \pm C_\pm r, \quad \text{as } r \rightarrow \pm\infty, \quad (4)$$

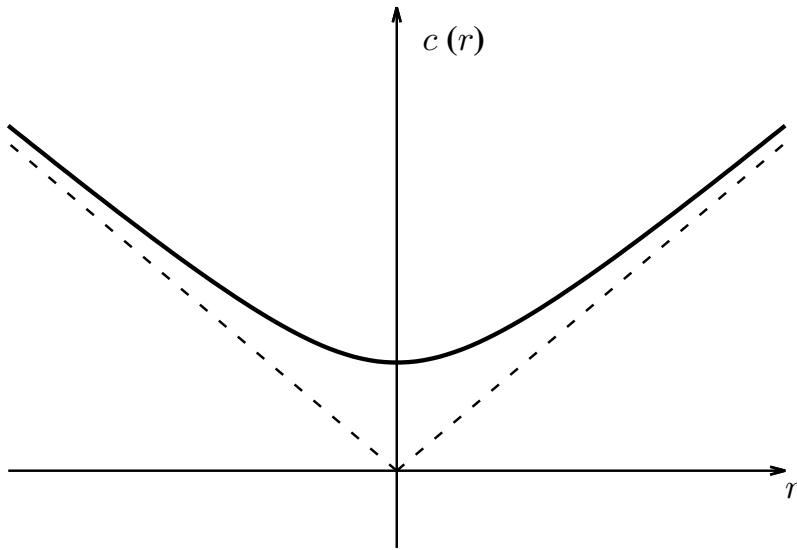


Figure 1: The behavior of the metric coefficient c .

where C_{\pm} are positive constants. For consistency, the Galileon field supposedly supporting a wormhole is also static and spherically symmetric, $\pi = \pi(r)$, and

$$\pi' \rightarrow 0 \quad \text{as } r \rightarrow \pm\infty, \quad (5)$$

where prime denotes d/dr .

Our main concern is stability, namely, the absence of ghosts and gradient instabilities in the Galileon perturbations about the solution $\pi(r)$. We observe that in any dimension, there is tension between the properties of the Galileon energy-momentum tensor that can support a wormhole, on the one hand, and stability requirement, on the other. In 3-dimensional space-time ($d = 1$) this tension is so strong that under very mild assumption on the asymptotic behavior of π' at spatial infinity, we show that there are no stable wormholes at all, the result somewhat reminiscent of Ref. [24]. We cannot prove similar no-go theorem in 4- or higher-dimensional space-time, but we will see that the simplest wormhole shapes are also inconsistent with stability. By a wormhole of the simplest shape we mean a solution for which dc/dR , where $R = \int bdr$ is the proper radial distance, monotonously increases¹ from -1 to 1 as R increases from $-\infty$ to $+\infty$, see Fig. 2. Similar property holds in the coordinate frame such that² $b(r) = a^{-1}(r)$, namely, if dc/dr monotonously increases in this frame as r increases, then the wormhole is unstable. These are our main results: under the assumptions

¹In fact, our observation is somewhat stronger: wormholes for which $|dc/dR| \leq 1$ at all R are inconsistent with stability.

²This frame can be called Schwarzschild.

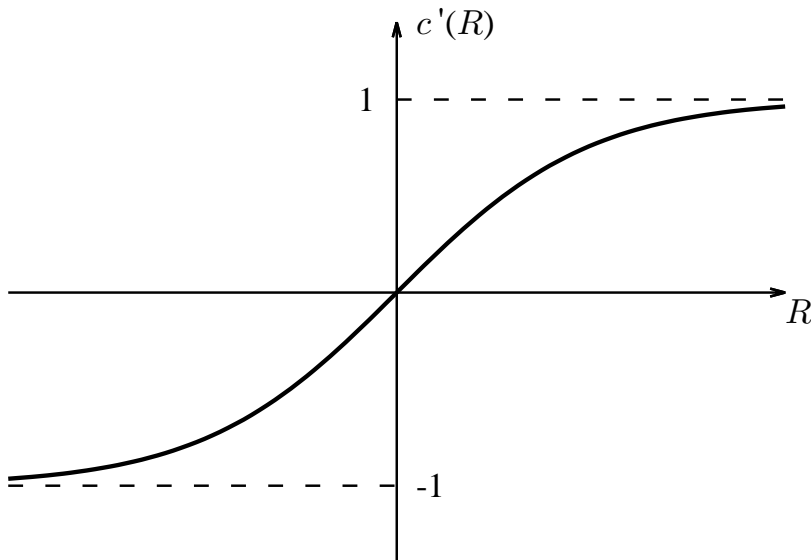


Figure 2: The behavior of dc/dR for a wormhole of the simplest shape. $R = \int bdr$ is the proper radial distance.

stated above, Galileons cannot support stable wormholes in 3-dimensional space-time, while in higher dimensions, stable Galileon-supported wormholes, at all exist, must have quite non-trivial properties.

The paper is organized as follows. We discuss the properties of the energy-momentum tensor that supports a wormhole in Section 2. In Section 3 we consider the static, spherically symmetric Galileons and present the form of the energy-momentum tensor and stability conditions of the Galileon perturbations. We derive our main results in Section 4 and conclude in Section 5.

2 Averaged NEC violation

Let us establish some features of the energy-momentum tensor supporting a wormhole. These are similar to the properties that lead to averaged NEC violation (ANEC violation), see Ref. [1] and references therein. To this end we recall that the point-wise NEC violation occurs when $T_\mu^\nu k^\mu k_\nu < 0$ for some null vector k^μ , while the ANEC violation in broad sense is the negative value of some line integral of $T_\mu^\nu k^\mu k_\nu$. In our context the relevant null vector has components $k^0 = a^{-1}$, $k^r = b^{-1}$ and $k^\alpha = 0$, so that $T_\mu^\nu k^\mu k_\nu = T_0^0 - T_r^r$; the line integrals we encounter have the form $\int_{-\infty}^{+\infty} dr \varphi(r) (T_0^0 - T_r^r)$ with a positive function $\varphi(r)$.

We set $8\pi G = 1$ and make use of the Einstein equations³ $T^\mu_\nu = G^\mu_\nu$, where

$$G^0_0 = d \left[\frac{c'b'}{cb^3} - \frac{c''}{cb^2} - \frac{d-1}{2} \left(\frac{c'^2}{c^2b^2} - \frac{1}{c^2} \right) \right] , \quad (6a)$$

$$G^r_r = -d \left[\frac{a'c'}{acb^2} + \frac{d-1}{2} \left(\frac{c'^2}{c^2b^2} - \frac{1}{c^2} \right) \right] , \quad (6b)$$

$$G^\alpha_\beta = \delta^\alpha_\beta G^\Omega , \quad (6c)$$

with

$$G^\Omega = - \left[\frac{a''}{ab^2} - \frac{a'b'}{ab^3} + (d-1) \left(\frac{c''}{cb^2} + \frac{c'a'}{cab^2} - \frac{c'b'}{cb^3} \right) + \frac{(d-1)(d-2)}{2} \left(\frac{c'^2}{c^2b^2} - \frac{1}{c^2} \right) \right] .$$

By combining Eqs. (6a) and (6b) one obtains

$$T^0_0 - T^r_r = -d \frac{a}{bc} \left(\frac{c'}{ab} \right)' . \quad (7)$$

The latter equation leads to the most commonly used form of the ANEC violation [1], namely,

$$\int_{-\infty}^{+\infty} dr \frac{b}{a} (T^0_0 - T^r_r) = -d \int_{-\infty}^{+\infty} dr \frac{c'^2}{abc^2} < 0$$

(the surface term appearing when integrating by parts vanishes because of the asymptotics (3) or (4)). This can be generalized to

$$\int_{-\infty}^{+\infty} dr \frac{bc^\alpha}{a} (T^0_0 - T^r_r) < 0 \quad \text{for all } \alpha \leq 1 . \quad (8)$$

The generalization is straightforward for $\alpha < 1$ (the surface term again vanishes), while for $\alpha = 1$ one has

$$\int_{-\infty}^{+\infty} dr \frac{bc}{a} (T^0_0 - T^r_r) = -d \left(\frac{C_+}{a_+} + \frac{C_-}{a_-} \right)$$

(where $C_\pm = 1$ for $d \geq 2$).

We will see that the Galileon energy-momentum tensor obeys $T^\alpha_\beta = \delta^\alpha_\beta T^\Omega$ with

$$T^\Omega = T^0_0 , \quad (9)$$

which implies $G^\Omega = G^0_0$ and gives

$$-\frac{c''}{b^2c} + \frac{c'b'}{b^3c} + \frac{a''}{b^2a} - \frac{a'b'}{b^3a} + (d-1) \frac{c'a'}{b^2ca} - (d-1) \left(\frac{c'^2}{b^2c^2} - \frac{1}{c^2} \right) = 0 .$$

³ $T^0_0 = \rho$, $T^r_r = -p_r$, $T^\alpha_\beta = -\delta^\alpha_\beta p_t$, where ρ is the energy density, p_r is the radial pressure and p_t is the tangential pressure. We will not use this nomenclature in what follows.

We use the latter equation to cast Eq. (7) into the following form,

$$T_0^0 - T_r^r = -\frac{d}{abc^{d-2}} \left(\frac{a'c^{d-2}}{b} \right)' - \frac{d(d-1)}{c^2} \left(1 - \frac{c'^2}{b^2} \right). \quad (10)$$

We will use this relation for $d \geq 2$. Assuming that

$$a'r^{d-2} \rightarrow 0 \quad \text{as } r \rightarrow \pm\infty, \quad (11)$$

we write

$$\begin{aligned} \int_{-\infty}^{+\infty} dr \, abc^{d-2} (T_0^0 - T_r^r) &= -d(d-1) \int_{-\infty}^{+\infty} dr \, abc^{d-4} \left(1 - \frac{c'^2}{b^2} \right) \\ &= -d(d-1) \int_{-\infty}^{+\infty} dR \, ac^{d-4} \left[1 - \left(\frac{dc}{dR} \right)^2 \right], \end{aligned} \quad (12)$$

where

$$R = \int b \, dr$$

is the proper radial distance. Provided the right hand side of eq. (12) is negative, this is another form of the ANEC violation. Note that the assumption (11) is not restrictive: once the total mass seen by an outside observer is finite, one has Newtonian asymptotics (recall that the number of space-time dimensions is $(d+2)$)

$$a = 1 + O(r^{-(d-1)}),$$

and therefore $a' = O(r^{-d})$.

3 Static, spherically-symmetric Galileons

3.1 Energy-momentum tensor

Turning to Galileons, the energy-momentum tensor of a theory with the Lagrangian (1) reads, in general,

$$T_{\mu\nu} = 2F_X \partial_\mu \pi \partial_\nu \pi + 2K_X \square \pi \cdot \partial_\mu \pi \partial_\nu \pi - \partial_\mu K \partial_\nu \pi - \partial_\nu K \partial_\mu \pi - g_{\mu\nu} F + g_{\mu\nu} g^{\lambda\rho} \partial_\lambda K \partial_\rho \pi,$$

where $F_\pi = \partial F / \partial \pi$, $F_X = \partial F / \partial X$, etc. (we reserve prime for d/dr), and $\partial_\mu K = K_\pi \partial_\mu \pi + 2K_X \nabla^\lambda \pi \nabla_\mu \nabla_\lambda \pi$. We immediately see from this expression that in the static spherically-symmetric case at hand, when $\pi = \pi(r)$, the energy-momentum tensor has the property (9).

We have, explicitly,

$$\begin{aligned} T_0^0 &= -F - K_\pi \left(\frac{\pi'}{b}\right)^2 + 2 \left(\frac{\pi'}{b}\right)^2 \frac{1}{b} \left(\frac{\pi'}{b}\right)' K_X , \\ T_r^r &= -2 \left(\frac{\pi'}{b}\right)^2 F_X - F + K_\pi \left(\frac{\pi'}{b}\right)^2 + 2K_X \left(\frac{\pi'}{b}\right)^3 \left(\frac{a'}{ba} + d\frac{c'}{bc}\right) . \end{aligned}$$

We are interested in the combination

$$\frac{1}{2} (T_0^0 - T_r^r) = \left(\frac{\pi'}{b}\right)^2 \left[F_X - K_\pi + K_X \frac{1}{b} \left(\frac{\pi'}{b}\right)' - K_X \frac{\pi'}{b} \left(\frac{a'}{ab} + d\frac{c'}{cb}\right) \right] , \quad (13)$$

which has to violate the ANECs of Sec. 2.

3.2 Stability conditions

We now turn to the discussion of the stability of Galileon perturbations about static, spherically symmetric backgrounds $\pi_c(r)$, and write $\pi = \pi_c + \chi$. We are interested in high momentum and frequency modes, so we concentrate on terms involving $\nabla_\mu \chi \nabla_\nu \chi$ in the quadratic Lagrangian or, equivalently, second order terms, proportional to $\nabla_\mu \nabla_\nu \chi$, in the linearized field equation. A subtlety here is that the Galileon field equation involves the second derivatives of metric, and the Einstein equations involve the second derivatives of the Galileon [18] (see also ref. [19]), and so do the linearized equations for perturbations. The trick is to integrate the metric perturbations out of the Galileon field equation by making use of the Einstein equations [18].

The full Galileon field equation reads

$$\begin{aligned} &(-2F_X + 2K_\pi - 2K_{X\pi} \nabla_\mu \pi \nabla^\mu \pi - 2K_X \square \pi) \square \pi + (-4F_{XX} + 4K_{X\pi}) \nabla^\mu \pi \nabla^\nu \pi \nabla_\mu \nabla_\nu \pi \\ &- 4K_{XX} \nabla^\mu \pi \nabla^\nu \pi \nabla_\mu \nabla_\nu \pi \square \pi + 4K_{XX} \nabla^\nu \pi \nabla^\lambda \pi \nabla_\mu \nabla_\nu \pi \nabla^\mu \nabla_\lambda \pi + 2K_X \nabla^\mu \nabla^\nu \pi \nabla_\mu \nabla_\nu \pi \\ &+ 2K_X R_{\mu\nu} \nabla^\mu \pi \nabla^\nu \pi + \dots = 0 ; \end{aligned}$$

hereafter dots denote terms without second derivatives. The subtle term is the last one here. The linearized equation can be written in the following form (hereafter we omit the subscript c in the notation for the Galileon background):

$$\begin{aligned} &-2[F_X + K_X \square \pi - K_\pi + \nabla_\nu (K_X \nabla^\nu \pi)] \nabla_\mu \nabla^\mu \chi \\ &-2[2(F_{XX} + K_{XX} \square \pi) \nabla^\mu \pi \nabla^\nu \pi - 2(\nabla^\mu K_X) \nabla^\nu \pi - 2K_X \nabla^\mu \nabla^\nu \pi] \nabla_\mu \nabla_\nu \chi \\ &+ 2K_X R_{\mu\nu}^{(1)} \nabla^\mu \pi \nabla^\nu \pi + \dots = 0 , \quad (14) \end{aligned}$$

where the terms without the second derivatives of χ are omitted, and $R_{\mu\nu}^{(1)}$ is linear in metric perturbations. We now make use of the Einstein equations $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}$, or

$$R_{\mu\nu} = T_{\mu\nu} - \frac{1}{d} g_{\mu\nu} T^\lambda_\lambda ,$$

linearize the energy-momentum tensor and obtain for the last term in eq. (14)

$$2K_X R_{\mu\nu}^{(1)} \nabla^\mu \pi \nabla^\nu \pi = -2K_X^2 \left[-\frac{2(d-1)}{d} X^2 \square \chi + 4X \nabla^\mu \pi \nabla^\nu \pi \nabla_\mu \nabla_\nu \chi \right] + \dots \quad (15)$$

The resulting linearized Galileon field equation is obtained from the following quadratic Lagrangian:

$$\begin{aligned} L^{(2)} = & [F_X + K_X \square \pi - K_\pi + \nabla_\nu (K_X \nabla^\nu \pi)] \nabla_\mu \chi \nabla^\mu \chi \\ & + [2(F_{XX} + K_{XX} \square \pi) \nabla^\mu \pi \nabla^\nu \pi - 2(\nabla^\mu K_X) \nabla^\nu \pi - 2K_X \nabla^\mu \nabla^\nu \pi] \nabla_\mu \chi \nabla_\nu \chi \\ & + \delta L^{(2)} , \end{aligned}$$

where $\delta L^{(2)}$ corresponds to the term (15):

$$\delta L^{(2)} = -\frac{2(d-1)}{d} K_X^2 X^2 \nabla_\mu \chi \nabla^\mu \chi + 4K_X^2 X \nabla^\mu \pi \nabla^\nu \pi \nabla_\mu \chi \nabla_\nu \chi .$$

If we did not set $8\pi G = 1$, the term $\delta L^{(2)}$ would contain a factor $8\pi G$, while the rest of the quadratic Lagrangian would be independent of G .

Specifying to static, spherically symmetric background, we find

$$L^{(2)} = a^{-2} \tilde{\mathcal{G}}^{00} \dot{\chi}^2 - b^{-2} \tilde{\mathcal{G}}^{rr} (\chi')^2 - c^{-2} \tilde{\mathcal{G}}^{\Omega} \gamma^{\alpha\beta} \partial_\alpha \chi \partial_\beta \chi ,$$

where the effective metric is

$$\tilde{\mathcal{G}}^{\mu\nu} = \mathcal{G}^{\mu\nu} + \delta \mathcal{G}^{\mu\nu} ,$$

with

$$\mathcal{G}^{00} = F_X - K_\pi - \frac{K'_X}{b} \frac{\pi'}{b} - 2K_X \frac{1}{b} \left(\frac{\pi'}{b} \right)' - 2dK_X \frac{c'}{cb} \frac{\pi'}{b} , \quad (16a)$$

$$\mathcal{G}^\Omega = F_X - K_\pi - \frac{K'_X}{b} \frac{\pi'}{b} - 2K_X \frac{1}{b} \left(\frac{\pi'}{b} \right)' - 2(d-1)K_X \frac{c'}{cb} \frac{\pi'}{b} - 2K_X \frac{a'}{ab} \frac{\pi'}{b} , \quad (16b)$$

$$\begin{aligned} \mathcal{G}^{rr} = & F_X - 2F_{XX} \left(\frac{\pi'}{b} \right)^2 - K_\pi + \frac{K'_X}{b} \frac{\pi'}{b} - 2K_X \frac{\pi'}{b} \left(\frac{a'}{ba} + d \frac{c'}{bc} \right) \\ & + 2K_{XX} \left(\frac{\pi'}{b} \right)^2 \frac{1}{b} \left(\frac{\pi'}{b} \right)' + 2K_{XX} \left(\frac{\pi'}{b} \right)^3 \left(\frac{a'}{ba} + d \frac{c'}{bc} \right) , \end{aligned} \quad (16c)$$

with $K'_X = dK_X/dr$ and

$$\begin{aligned} \delta \mathcal{G}^{00} = \delta \mathcal{G}^\Omega = & -\frac{2(d-1)}{d} K_X^2 \left(\frac{\pi'}{b} \right)^4 \\ \delta \mathcal{G}^{rr} = & \frac{2(d+1)}{d} K_X^2 \left(\frac{\pi'}{b} \right)^4 . \end{aligned}$$

The stability conditions for the perturbations are

$$\tilde{\mathcal{G}}^{00} > 0, \quad \tilde{\mathcal{G}}^{rr} \geq 0, \quad \tilde{\mathcal{G}}^\Omega \geq 0. \quad (17)$$

There are no ghosts and/or gradient instabilities only if these conditions are satisfied.

Since both $\delta\mathcal{G}^{00}$ and $\delta\mathcal{G}^\Omega$ vanish in the case of three-dimensional space-time ($d = 1$) and are negative in higher dimensions, the first and third stability conditions in eq. (17) imply

$$\mathcal{G}^{00} > 0, \quad \mathcal{G}^\Omega > 0. \quad (18)$$

We will see that these necessary conditions, taken together with the ANEC violation, are difficult to satisfy.

We note in passing that at least in theories which have conventional and stable scalar field theory limit as $\partial\pi \rightarrow 0$, namely, $F \rightarrow X - V(\pi)$ (and $K \rightarrow 0$, see eq. (2)), there is unavoidable superluminality near this limit. Indeed, by choosing the background with $\dot{\pi} = \partial_\alpha \pi = 0$ at a given moment of time and setting $a = b = 1$, $c = r$ one can always have, with an appropriate sign of π' ,

$$\tilde{\mathcal{G}}^\Omega - \tilde{\mathcal{G}}^{00} = 2K_X \frac{\pi'}{r} > 0,$$

which means that the modes normal to the radial direction propagate superluminally. At small π' such a background is stable, since in this regime $\tilde{\mathcal{G}}^{00} = \tilde{\mathcal{G}}^\Omega = \tilde{\mathcal{G}}^{rr} = F_X$ modulo small corrections. This observation is in line with earlier results on the superluminality of Galileons [23].

4 Tensions, No-Go's

4.1 Generalities

Let us see that the necessary conditions for the stability, eq. (18), are in tension with the ANEC violation discussed in Sec. 2. To this end, we multiply \mathcal{G}^{00} given by eq. (16a) by $\mu \cdot (\pi'/b)^2$, where $\mu(r)$ is yet unspecified positive function, integrate over r from $-\infty$ to $+\infty$, and integrate by parts the third term in the right hand side of eq. (16a), setting (recall our assumption, eq. (5))

$$\pi'^3 K_X \mu \rightarrow 0 \quad \text{as } r \rightarrow \pm\infty. \quad (19)$$

We obtain

$$\begin{aligned} & \int_{-\infty}^{+\infty} dr \, \mu(r) \left(\frac{\pi'}{b} \right)^2 \mathcal{G}^{00} \\ &= \int_{-\infty}^{+\infty} dr \, \mu(r) \left(\frac{\pi'}{b} \right)^2 \left[F_X - K_\pi + K_X \frac{1}{b} \left(\frac{\pi'}{b} \right)' + K_X \frac{\pi'}{b} \frac{1}{b} \left(\frac{\mu'}{\mu} - \frac{b'}{b} - 2d \frac{c'}{c} \right) \right] > 0. \end{aligned} \quad (20)$$

We now choose

$$\mu = \frac{bc^d}{a}$$

and see that the integrand in the right hand side of eq. (20) is proportional to $(T_0^0 - T_r^r)$ given by eq. (13). Thus,

$$\int_{-\infty}^{+\infty} dr \frac{bc^d}{a} (T_0^0 - T_r^r) > 0 . \quad (21)$$

This is in tension with eq. (8), although for $d \geq 2$ there is no direct contradiction.

By performing the same procedure with \mathcal{G}^Ω with the measure

$$\mu = abc^{d-2}$$

we obtain

$$\int_{-\infty}^{+\infty} dr abc^{d-2} (T_0^0 - T_r^r) > 0 . \quad (22)$$

More generally, we consider a combination

$$(1 - \beta)\mathcal{G}^{00} + \beta\mathcal{G}^\Omega > 0 \quad 0 \leq \beta \leq 1 ,$$

choose the measure as

$$\mu = a^{2\beta-1}bc^{d-2\beta}$$

and find

$$\int_{-\infty}^{+\infty} dr a^{2\beta-1}bc^{d-2\beta} (T_0^0 - T_r^r) > 0 \quad \text{for all } 0 \leq \beta \leq 1 , \quad (23)$$

which should hold together with the ANEC violation inequality (8). Obviously, there is tension between these inequalities. As an example, for $d = 2$ (4-dimensional space-time) one can choose $\alpha = 1$ in eq. (8) and $\beta = 1/2$ in eq. (23) to get

$$\begin{aligned} \int_{-\infty}^{+\infty} dr \frac{bc}{a} (T_0^0 - T_r^r) &< 0 , \\ \int_{-\infty}^{+\infty} dr bc (T_0^0 - T_r^r) &> 0 . \end{aligned}$$

This shows that the Galileon-supported wormholes, if any, must be quite tricky.

We note that our assumption (19) is very mild. The large- $|r|$ behavior of the relevant measures is at most $|r|^d$, so we need $\pi' = o(|r|^{-d/3})$. On the other hand, if the Galileon becomes an ordinary scalar field in the weak-field limit, one has $\pi' \propto |r|^{-d}$, which is more than sufficient. In other words, to violate the assumption (19) and at the same time have the Minkowski limit at $\partial\pi = 0$, the Galileon must be pretty contrived.

4.2 3-dimensional space-time

In the case of 3-dimensional space-time we have $d = 1$, so there is direct contradiction between the inequalities (8) with $\alpha = 1$ and (21). So, in this case we have a no-go theorem stating that there are no stable, static, spherically symmetric wormholes in Galileon theories with the Lagrangians of the form (1) and Minkowski limit at $\partial\pi = 0$. The only way to get around this theorem is to violate the property (19), which reads

$$r\pi'^3 K_X \rightarrow 0 \quad \text{as } r \rightarrow \pm\infty .$$

Attempting to explore this loophole is not promising, we think.

4.3 Space-times of more than 3 dimensions

For $d \geq 2$, including the most physically interesting case of 4-dimensional space-time ($d = 2$), there is no direct contradiction between the ANEC violation inequality (8) and stability inequality (23). Yet the possible shapes of wormholes are strongly constrained. To this end, we first consider eqs. (12) and (22). Taken together, these rule out wormholes for which

$$\left| \frac{dc}{dR} \right| \leq 1 \quad \text{for all } r ,$$

including wormholes with monotonous $dc(R)/dR$, Fig. 2.

Another constraint follows directly from eq. (7). Namely, consider the often used (“Schwarzschild”) coordinate frame, in which

$$b(r) = a^{-1}(r) .$$

The function $c'(r)$ cannot be monotonous in this frame either. Indeed, if c' monotonously increases in this frame from -1 to 1 as r runs from $-\infty$ to $+\infty$, then $c'' > 0$, and $(T_0^0 - T_r^r) < 0$ everywhere. This contradicts any of the inequalities (23).

These two properties rule out the simplest wormhole shapes.

To conclude this Section we note that adding conventional matter that does not violate the NEC would not help. The ANEC violation inequalities of Sec. 2 must hold for the total energy-momentum tensor, and since the conventional matter has $T_0^0 - T_r^r > 0$, these inequalities must still be valid for the Galileon contribution to the total T_ν^μ . Thus, our analysis remains intact.

5 Discussion

Even though our findings are not completely conclusive, they show that constructing Galileon-supported wormholes must be tricky, if at all possible. One way to get around of our constraints would be to give up our initial assumption that the Minkowski regime occurs at

$\partial\pi = 0$, which lead to eq. (5). This is indeed a possibility if $F = F(X)$, $K = K(X)$ depend on $X = (\partial\pi)^2$ but not on π itself. Then the linear Galileon background, say, $\pi = Qx^1$ where Q is a constant, obeys the Galileon field equation and has vanishing energy-momentum tensor provided that $F(-Q^2) = 0$ and $F_X(-Q^2) = 0$, somewhat resembling the ghost condensate case [25]. In our spherically symmetric setting, the Galileon with asymptotics $\pi \rightarrow \pm Qr$ as $r \rightarrow \pm\infty$ would violate eq. (19), so our arguments would not work. Unlike in the ghost condensate case, however, the dispersion relation for perturbations about $\pi = Qx^1$ at quadratic level in momenta and frequency is $(p^1)^2 = 0$. As discussed in Ref. [26], this is problematic from the effective field theory viewpoint: genuine higher order terms would modify the dispersion relation to

$$(p^1)^2 = A \frac{\omega^4}{\Lambda^2},$$

where Λ is a UV cutoff, and A is generically of order one. This would yield the gradient instability with the time scale as small as Λ^{-1} . Nevertheless, one can assume that A is fine tuned to be very small, so searching for stable wormholes with $\pi \rightarrow \pm Qr$ at large $|r|$ is of interest.

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